

Applications of Fourier Transform in Engineering Field

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ABSTRACT: Fourier Transform is useful in the study of solution of partial differential equation to solve initial boundary value problems. A Fourier Transform when applied to partial differential equation reduces the number of independent variables by one. We use Fourier Transform in signal & image processing. It is also useful in cell phones, LTI system & circuit analysis

KEYWORDS: Fourier Transform, Inverse Fourier Transform, Discrete Fourier Transform (DFT)

I. INTRODUCTION

We obtain Fourier Transform by a limiting process of Fourier series. Since it was first used by French Mathematician Jean Baptiste Fourier (1768-1830) in a manuscript submitted to the Institute of France in 1807. He said that Fourier Transform is a mathematical procedure which transforms a function from time domain to frequency domain. Fourier analysis is useful in almost every aspect of the subject ranging from solving LDE to developing computer models, to the processing & analysis of data. The Fourier Transform is a magical mathematical tool that decomposes any function into the sum of sinusoidal basis functions. The Fourier Transform is a tool that breaks a waveform (a function or signal) into an alternate representation characterized by sine & cosines.

Definition of Fourier Transform –

The Fourier Transform is a generalization of the Fourier series. It only applies to continuous & a periodic functions.

We defined Fourier Transform of a piecewise continuous & absolutely integrable function $x(t)$ by

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

Inverse Fourier Transform-

We define inverse Fourier Transform by using Fourier Transform

$$X(t) = F^{-1}\{X(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} d\omega$$

Discrete Fourier Transform-

Let $x[n]$ be a finite-length sequence of length N i.e. $x[n] = 0$ outside the range $0 \leq n \leq N-1$

The Discrete Fourier Transform of $x[n]$, denoted as $X[k]$, is defined by

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$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, k=0,1,\dots,N-1$$

Where W_N is the N^{th} root of unity given by

$$W_N = e^{-j\left(\frac{2\pi}{N}\right)}$$

The inverse discrete Fourier transform is given by

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}, n=0,1,\dots,N-1$$

The Discrete Fourier Transform is closely related to Discrete Fourier series & the Fourier Transform. The Discrete Fourier transform is the appropriate Fourier representation for digital computer realization because it is discrete and of finite length in both time and frequency domain.

Also the Fast Fourier Transform computes DFT & produces exactly the same result as evaluating DFT definition directly. It is much faster than DFT.

Properties of Fourier Transform

Properties of the Fourier transform facilitate the transformation from the time domain to frequency domain & vice versa.

1. Linearity-

The Fourier Transform satisfies linearity & principle of superposition

Consider two functions $x_1(t)$ & $x_2(t)$

If $F[x_1(t)] = X_1(\omega)$, $F[x_2(t)] = X_2(\omega)$

Then $F[a_1 x_1(t) + a_2 x_2(t)] = a_1 X_1(\omega) + a_2 X_2(\omega)$

2. Scaling –

$F[x(t)] = X(\omega)$

If a is real constant then

$$F[x(at)] = \frac{1}{|a|} X(\omega)$$

3. Symmetry-

If $F[x(t)] = X(\omega)$ then $X(t) = 2\pi X(-\omega)$

4. Convolution –

Fourier transform makes the convolution of 2 signals into the product of their Fourier Transforms. There are two types of convolution properties, one for time domain & other for frequency domain.

a) Time convolution –

If $F[x_1(t)] = X_1(\omega)$, $F[x_2(t)] = X_2(\omega)$

Then $Y(\omega) = X_1(\omega) \cdot X_2(\omega)$

b) Frequency convolution –

$$x_1(t) \cdot x_2(t) = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$

5. Shifting Property –

$$x(t - t_0) = e^{-j\omega t_0} X(\omega)$$

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6. Duality –

$$X(\omega) = 2\pi x(-\omega)$$

7. Differentiation –

$$\frac{dx(t)}{dt} = j\omega X(\omega)$$

8. Modulation Property-

$$F\{x(t) \cos at\} = \frac{1}{2}\{X(\omega+a) + X(\omega-a)\}$$

$$F\{x(t) \sin at\} = \frac{1}{2j}\{X(\omega+a) - X(\omega-a)\}$$

Relation between Fourier Transform & Laplace Transform-

We know that

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

By using Laplace Transform,

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$$

Here we can say that Fourier Transform is a special case of Laplace Transform in which

$$s = j\omega$$

i.e. $X(s)_{s=j\omega} = F\{x(t)\}$.

II. APPLICATIONS OF FOURIER TRANSFORM

The Fourier Transform method is applicable in many fields of science & technology such as

- 1) Application to IBVP
- 2) Circuit Analysis
- 3) Signal Analysis
- 4) Cell phones
- 5) Image Processing
- 6) Signal Processing & LTI system

Now we take brief overview of these applications

1. Application to Initial boundary value problems (IBVP) –

The solution of a IBVP consists of a partial differential equation together with boundary & initial conditions can be solved by Fourier Transform method. Here we solve the heat equation analytically by using boundary condition. In this case partial differential equations reduce to an Ordinary Differential Equations in Fourier Transform which is solved.

Now see the example

Example – Heat equation in one spatial dimension.

$$\frac{\partial T}{\partial t} = p \frac{\partial^2 T}{\partial x^2}$$

Where p is thermal diffusivity.

Open boundaries – $T(x, t)$ defined on

$$-\infty < x < +\infty \text{ and } t \geq 0$$

Also, require that $T(x, t) \rightarrow 0$ as $x \rightarrow \pm\infty$

Initial value problem: $T(x, t=0) = \phi(x)$

Solution- Apply Fourier Transform to heat equation (at constant t)

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$$F [T_x] = F[p T_{xx}] \dots\dots\dots 1$$

$$\frac{\partial}{\partial t} F[T] = -k^2 p F[T] \dots\dots\dots 2$$

Let us denote $r(k, t) = F[T(x, t)]$

$$\frac{\partial}{\partial t} r(k, t) = -k^2 p r(k, t) \dots\dots\dots 3$$

Equation 3 is now a simple ordinary differential equation .

Heat equation is much easier to solve in the Fourier domain.

The solution is

$$r(k, t) = e^{-k^2 p t} r(k, 0) \dots\dots\dots 4$$

Still need to transform the initial condition

$$T(x, 0) = \phi(x)$$

$$F[\phi(x)] = F[T(x, t=0)] = r(k, 0) \dots\dots\dots 5$$

Combining equations 4 & 5

$$r(k, t) = e^{-k^2 p t} F[\phi(x)] \dots\dots\dots 6$$

In order to obtain solution in real space ,apply inverse Fourier Transform

$$T(x, t) = F^{-1}[r(k, t)] \\ = F^{-1}[e^{-k^2 p t} F[\phi(x)]] \dots\dots\dots 7$$

However , we use convolution theorem on right hand side .

I recall this

$$F[f * g] = F(f).F(g)$$

Therefore ,

$$f * g = F^{-1}[F(f).F(g)] \dots\dots\dots 8$$

Now we apply this to equation 7

$$\text{Let } F(f) = e^{-k^2 p t} \text{ and } F(g) = F[\phi(x)].$$

It follows that $g = F^{-1}\{F[\phi(x)]\} = \phi(x)$ by definition.

$$f = F^{-1}[e^{-k^2 p t}] \dots\dots\dots 9$$

$$= \frac{1}{\sqrt{2 p t}} e^{-\frac{x^2}{4 p t}} \dots\dots\dots 10$$

In last step we used inverse Fourier transform

Of a Gaussian.

$$\text{Since } T(x, t) = f * g \dots\dots\dots 11$$

According to equations 7 & 8 we have

$$T(x, t) = \frac{1}{\sqrt{2 p t}} e^{-\frac{x^2}{4 p t}} * \phi(x) \dots\dots\dots 12 \\ = \frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2 p t}} e^{-(x-c)^2 / 4 p t} . \phi(c) dc \dots\dots 13 \\ = \frac{1}{\sqrt{4 \pi p t}} \int_{-\infty}^{\infty} e^{-(x-c)^2 / 4 p t} . \phi(c) dc$$

This is the fundamental solution of the heat equation with open boundaries for an initial condition $T(x, t = 0) = \phi(x)$

2.Circuit Analysis-

There are many linear circuits used in Electronic engineering field .These circuits include various components like capacitor, inductor ,resistor etc. Every Electronic circuit can be modelled using mathematical equations.

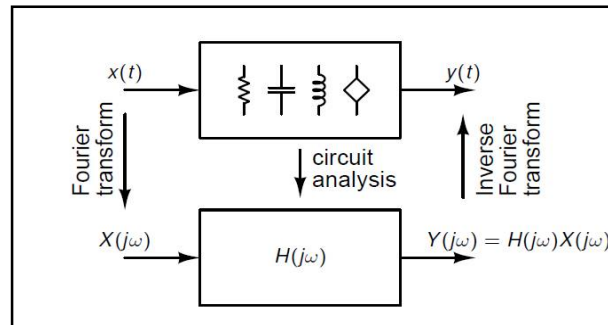
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See this diagram



Where $x(t)$ - actual signal applied as input to the circuit.

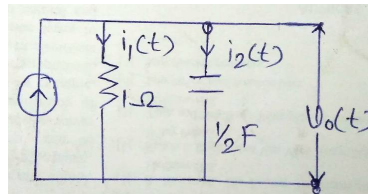
$y(t)$ – output of the circuit

Now to perform frequency analysis of the circuit Fourier Transform is used.

Here we take one example.

In this example we have to find output voltage $v_0(t)$ by using Fourier transform.

Solution -



Where $i(t)$ = current source

$i_1(t)$ = current flowing through resistance

$i_2(t)$ = current flowing through capacitor

$v_0(t)$ = output voltage

$i(t) = e^{-t}u(t)$

According to Kirchoff's law

$i(t) = i_1(t) + i_2(t)$, $v_0(t) = i_2(t)$ and

$e^{-t} u(t) = v_0(t) + \frac{1}{2}v_0(t)$

By taking Fourier transform

$$\frac{1}{j\omega+1} = v_0(j\omega) + \frac{1}{2}(j\omega) v_0(j\omega)$$

$$\frac{1}{j\omega+1} = v_0(j\omega) \left[1 + \frac{1}{2}j\omega \right]$$

$$\frac{1}{j\omega+1} = v_0(j\omega) \left[\frac{2+j\omega}{2} \right]$$

$$v_0(j\omega) = 2 \left[\frac{1}{(j\omega+1)(2+j\omega)} \right] \dots\dots\dots 1$$

By using partial fraction method,

$$\frac{1}{(j\omega+1)(2+j\omega)} = \frac{A}{(j\omega+1)} + \frac{B}{(2+j\omega)} \dots\dots\dots 2$$

$$\frac{1}{(j\omega+1)(2+j\omega)} = \frac{A(2+j\omega)+B(j\omega+1)}{(j\omega+1)(2+j\omega)}$$

$$1 = A(2+j\omega) + B(j\omega+1) \dots\dots 3$$

Put $j\omega + 1 = 0 \rightarrow j\omega = -1$ in equation 3

we get $A = 1$.

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Similarly if we put $2 + j\omega = 0 \rightarrow j\omega = -2$ in equation 3 we get $B = -1$.
Therefore equation 2 becomes

$$\frac{1}{(j\omega+1)(2+j\omega)} = \frac{1}{(j\omega+1)} + \frac{-1}{(2+j\omega)}$$

Put this in equation 1 we get

$$v_0(j\omega) = 2 \left[\frac{1}{(j\omega+1)} + \frac{-1}{(2+j\omega)} \right]$$

Taking Inverse Fourier transform we get

$$F^{-1}[v_0(j\omega)] = 2F^{-1} \left\{ \left[\frac{1}{(j\omega+1)} + \frac{-1}{(2+j\omega)} \right] \right\}$$

$$v_0(t) = 2 [e^{-t} u(t) - e^{-2t} u(t)]$$

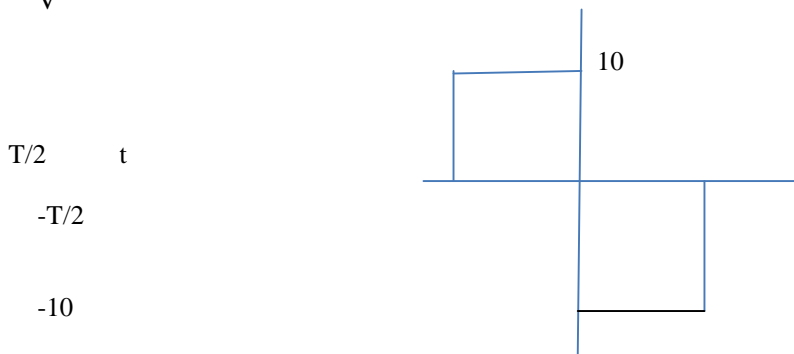
Since Fourier Transform helps us to analyse the behavior of circuit when different inputs are applied.

3.Signal Analysis-

Signal is the important part of any electronic circuit to design & analyze various electronic circuits. It is necessary to do the signal analysis. Now I take example related to signals.

Here we have to find the magnitude and phase spectrum of the waveform shown in the figure below.

V



Solution – The equation of voltage waveform is given by

$$V(t) = \begin{cases} 10 & -T/2 \leq t \leq 0 \\ -10 & 0 \leq t \leq T/2 \\ 0 & \text{o.w} \end{cases}$$

$$\begin{aligned} V(\omega) &= F[v(t)] = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt \\ &= \int_{-T/2}^0 v(t) e^{-j\omega t} dt + \int_0^{T/2} v(t) e^{-j\omega t} dt \\ &= \int_{-T/2}^0 10 e^{-j\omega t} dt + \int_0^{T/2} -10 e^{-j\omega t} dt \\ &= \int_{-T/2}^0 10 e^{-j\omega t} dt + \int_0^{T/2} -10 e^{-j\omega t} dt \\ &= \int_0^{T/2} 10 e^{j\omega t} dt - \int_0^{T/2} 10 e^{-j\omega t} dt \end{aligned}$$

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$$= 10 \int_0^T (e^{j\omega t} - e^{-j\omega t}) dt$$

$$= 10 \times 2j \int_0^{T/2} \sin \omega t dt$$

$$= 20j \left[\frac{-\cos \omega t}{\omega} \right]_0^{T/2}$$

$$V(\omega) = \frac{20j}{\omega} \left[1 - \frac{\cos \omega T}{2} \right]$$

$$\text{Magnitude of spectrum} = \frac{20}{\omega} \left[1 - \frac{\cos \omega T}{2} \right]$$

$$\text{Also } \phi(\omega) = \tan^{-1} \left[\frac{\text{Im}v(\omega)}{\text{Re}v(\omega)} \right]$$

$$= \tan^{-1} \left[\frac{20(1 - \cos \frac{\omega T}{2})}{0} \right]$$

$$= \begin{cases} \pi/2 & \omega is + ve \\ -\pi/2 & \omega is - ve \end{cases}$$

To find out frequency components in the given signal Fourier Transform is used.

4. Cell phones-

Communication is all based on Mathematics .The communication includes automatic transmission of data over wires and radio circuits through signals .Cell phones are one of the most prominent communication device , the cell phone is dramatically changing the way people interact and communicate with each other.

The principle of Fourier Transform is used in signal ,such as sound produced by a musical instrument For e.g- piano, violin ,drum any sound recording can be represented as the sum of a collection of sine and cosine waves with various frequencies and amplitudes. This collection of waves can then be manipulated with relative ease. Our mobile phone has performing Fourier Transform. Every mobile device – such as netbook, tablet ,and phone have been built in high speed cellular connection , just like Fourier Transform. Humans very easily perform it mechanically everyday. For ex. When you are in a room with a great deal of noise & you selectively hear your name above the noise, then you just performed Fourier transform.

5. Image Processing-

Fourier transform is used in a wide range of applications such as image analysis ,image filtering , image reconstruction and image compression.

The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components.

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The Fourier Transform is used if we want to access the geometric characteristics of a spatial domain image. Because the image in the Fourier domain is decomposed into its sinusoidal components, it is easy to examine or process certain frequencies of the image, thus influencing the geometric structure in the spatial domain.

In most implementations the Fourier image is shifted in such a way that the DC-value (*i.e.* the image mean) $F(0,0)$ is displayed in the center of the image. The further away from the center an image point is, the higher is its corresponding frequency.

6. Signal Processing & LTI system-

The Fourier Transform is extensively used in the field of Signal Processing. In fact, the Fourier Transform is probably the most important tool for analyzing signals in that entire field.

A signal is any waveform (function of time). This could be anything in the real world - an electromagnetic wave, the voltage across a resistor versus time, the air pressure variance due to your speech (*i.e.* a sound wave), or the value of Apple Stock versus time. Signal Processing then, is the act of processing a signal to obtain more useful information, or to make the signal more useful.

Suppose we have a box that accepts an input signal and produces an output signal from that. Such a box can be thought of as a system:



A System which takes an input signal and produces an output signal

when we view the Fourier Transform of the output, we now know how the system reacts to every possible frequency. The reason for this goes back to the linearity of the Fourier Transform: the impulse in time can be thought of as an infinite sum of sinusoids at every possible frequency. The output result then is the sum of the responses to each frequency.

Fourier Transform visualizes the effect of an LTI system simply and the analysis much easier.

The Fourier Transform is extensively used in LTI system theory, filtering and signal processing. In fact, the majority of the analysis takes place in the frequency domain, making the understanding of Fourier Theory indispensable.

III. CONCLUSION

In this paper we can say that The Fourier Transform resolves functions or signals into its mode of vibration. It is used in designing electrical circuits, solving differential equations, signal processing, signal analysis, image processing & filtering.

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